tures of phase transition and vanishing of the first anisotropy constant may be significant.

To summarize, it can be said that in 3d-ferromagnetics and a number of compounds of the 3d-metals there exists an isotropic magnetic phase, the thermodynamic properties of which are fully defined by exchange interaction. Relativistic (spin-orbital, dipole-dipole) interactions manifest themselves in a cooperative manner, leading to appearance of magnetic anisotropy and distortion of the crystal at temperatures T_{LS} which do not coincide with the Curie temperature. The above is supported by the behavior of $K_1(T)$, $K_2(T)$, and DTA curves of the ferromagnetics studied.

A number of questions related to theoretical group analysis of relativistic transitions and construction of phase diagrams are beyond the scope of the present study, but will be examined in another effort.

NOTATION

T_C, Curie temperature; T_1, temperature at which first magnetic anisotropy constant vanished; T_{LS}, temperature of relativistic (spin-orbital) phase transition: M, spontaneous magnetization vector; C, pseudovector order parameter; C, absolute value of pseudovector order parameter; Φ, Landau thermodynamic potential; A_1, A_2, A_3, B_1, B_2, coefficients in expansion of thermodynamic potential; m, unit vector directed along spontaneous magnetization vector; m_x, m_y, m_z, components of unit vector m; K_1, K_2, first and second magnetic anisotropy constants of cubic ferromagnetic; Fe_3O_4, chemical formula of magnetite.

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TEMPERATURE FIELD INHOMOGENEITY ON THE SURFACE OF AN ELECTROLYTIC HYGROMETER

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Determination of the temperature field on a hygrometer surface is reduced to measurement of electrical quantities by solution of the thermal conductivity problem for the hygrometer body. Differences in results from direct experimental data on surface temperature obtained by other authors using hygrometer models are analyzed.

Electrolytic heated hygrometers have now been widely used for almost three decades to measure parameters characterizing the moisture content of gases [1]. Their advantages over other sensors for use as primary sensors in automated data collection and control systems are well known. A large number of publications dedicated to analysis of operation of these devices have studied their basic static and dynamic characteristics and offered mathematical models [2, 3]. However the fact remains that the known literature still lacks a detailed quantitative analysis of certain basic well known unique features of these devices.

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The present study will consider the causes producing an excess in the temperature of an electrolytic heated hygrometer above the three-phase equilibrium temperature of the salt used in the device. According to [4] the temperature elevation of a sensor of the most widely used construction comprises 0.3-0.8 K. The existence of such a temperature head in hygrometers of current design can easily be shown by comparing calibration characteristics of sensors produced by various firms with the three-phase equilibrium curve of the salt used.

An experimental study of hygrometer temperature elevation relative to the three-phase equilibrium temperature of the corresponding salt was carried out in [5], the authors of which experimentally detected and studied nonuniformity in the temperature field on the surfaces of model hygrometers. The measurements were performed with glass-encased thermocouples on hygrometer models with interelectrode distance increased by a factor of 4-8 times. Through analysis of their measurement data the authors concluded that the temperature distribution in the interelectrode gap of the model hygrometer was parabolic in form, with the temperature difference between the electrodes and the midpoint of the gap decreasing with decrease in gap spacing, comprising 3-7 K for the models studied. By extrapolating the experimental data of [5] to interelectrode distances occurring in real hygrometer constructions (of the order of 1 mm) we obtain the temperature distribution in the interelectrode gap (Fig. 1).

For a more detailed analysis of the results of [5], we will use the latter to define the heat liberation distribution in the surface layer of the hygrometer. This can be done by formulating the problem of defining the thermal flux density on the hygrometer surface as a converse boundary problem for the thermal conductivity equation describing the process within the converter body. In view of the symmetry of the construction and hygrometer temperature field it is sufficient to consider a cylindrical section with height one half of the interelectrode distance. The axial symmetry of the temperature field in this section of the body allows us to neglect the angular coordinate. To formulate the problem we will require information on the thermophysical characteristics of the materials from which the hygrometer is constructed. Since we are dealing with a steady state temperature field it is sufficient to know the thermal conductivity of the materials. The similar thermal conductivities of materials used in multicomponent hygrometer construction allows us to treat the hygrometer material as homogeneous with an effective thermal conductivity of $\lambda = 0.3$ W/(m·K) with satisfactory accuracy.

As a boundary condition on the hygrometer surface we specify a condition of the first sort, namely, the temperature of that surface, extrapolated from the experimental data of [5] to a hygrometer with interelectrode distance 1 mm (see Fig. 1). With these considerations the mathematical formulation of the problem appears as follows:

steady state thermal conductivity equation in cylindrical coordinates

$$\frac{\partial^2 t(r,\ z)}{\partial r^2} + \frac{1}{r}\frac{\partial t(r,\ z)}{\partial r} + \frac{\partial^2 t(r,\ z)}{\partial z^2} = 0,\ 0 \leqslant r \leqslant R,\ 0 \leqslant z \leqslant h; \tag{1}$$

temperature field symmetry in hygrometer relative to midpoint of interelectrode gap, electrode, and axis of hygrometer cylinder

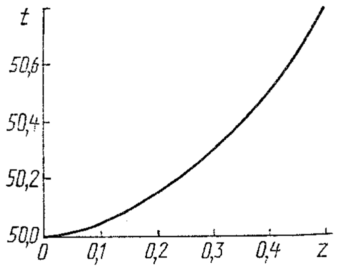

Fig. 1. Hygrometer surface temperature f(z) from Eq. (4) at $k = 3.2 \cdot 10^6$ K/m^2, extrapolated from experimental data of [5]. z, distance from midpoint of electrode gap, mm. t, °C.

$$\left.\frac{\partial t(r,z)}{\partial z}\right|_{z=0} = \left.\frac{\partial t(r,z)}{\partial z}\right|_{z=h} = 0; \tag{2}$$

$$\left.\frac{\partial t(r,z)}{\partial r}\right|_{r=0} = 0; \tag{3}$$

temperature condition on hygrometer surface

$$t(R,\ z) = f(z) = t_0 + kz^2. \tag{4}$$

The unknown is the thermal flux density on the hygrometer surface q(R, z), which is related to the temperature field thereon by the expression

$$q(R,\ z) = -\lambda \left.\frac{\partial t(r,z)}{\partial r}\right|_{r=R}. \tag{5}$$

Using the common method of solving boundary problems by separation of variables [6], for the temperature field in the body of the hygrometer cylinder we obtain an expression in the form of an infinite series

$$t(r,\ z) = t_0 + \frac{kh^2}{3} + \frac{4kh^2}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n}{I_0\left(\frac{n\pi}{h}R\right)} I_0\left(\frac{n\pi}{h}r\right)\cos\left(\frac{n\pi}{h}z\right). \tag{6}$$

Substituting Eq. (6) in Eq. (5) and differentiating the infinite series thus obtained, we write for the unknown thermal flux density the following expression:

$$q(R,\ z) = \frac{4k\lambda h}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\,\frac{I_1\left(\frac{n\pi}{h}R\right)}{I_0\left(\frac{n\pi}{h}R\right)}\cos\left(\frac{n\pi}{h}z\right). \tag{7}$$

As is evident from Eq. (7) the converse character of the problem of Eqs. (1)-(5), which consists of defining a boundary condition of the second sort, Eq. (5), in terms of a boundary condition of the first sort, Eq. (4) causes the solution to be expressed by a series with poor convergence. However, noting that the argument of the modified Bessel functions in Eq. (7) for our case ($h = 5\cdot 10^{-4}$ m, $R = 3.5\cdot 10^{-3}$ m) takes on only values greater than 22, and making use of the well known [7] asymptotic representation for these functions

$$I_\nu(z) \approx \frac{e^z}{\sqrt{2\pi z}}\left\{1 - \frac{\mu-1}{8z} + \frac{(\mu-1)(\mu-9)}{2!\,(8z)^2} - \frac{(\mu-1)(\mu-9)(\mu-25)}{3!\,(8z)^3} + \dots\right\},$$

on that in Eq. (7) the ratio $I_1(\frac{n\pi}{h}R)/\ I_0(\frac{n\pi}{h}R)$, which even at n = 1 has a value of 0.98, tends rapidly to unity with increase in n. Therefore, with no risk of severely distorting the result, in Eq. (7) we replace this ratio by unity and obtain an approximate expression for the thermal flux density

$$q(R,\ z) \approx \frac{4k\lambda h}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\cos\left(\frac{n\pi}{h}z\right). \tag{8}$$

To sum series (8) we may use known methods for improving convergence of a series [8]. In particular, using the well known relationship

$$\sum_{n=1}^{\infty}\frac{\cos nx}{n} = -\ln\left(2\sin\frac{x}{2}\right),$$

for the sum of Eq. (8) we obtain

$$q(R,\ z) \approx \frac{2k\lambda h}{\pi}\ln\left\{4\sin\left[\frac{\pi(h+z)}{2h}\right]\sin\left[\frac{\pi(h-z)}{2h}\right]\right\}. \tag{9}$$

Figure 2 shows thermal flux density distribution on the surface of a hygrometer as given by Eq. (9). Also shown there is the form of the function q(R, z) obtained by direct summation of series (7) by computer using computation precautions demanded by the slow convergence.

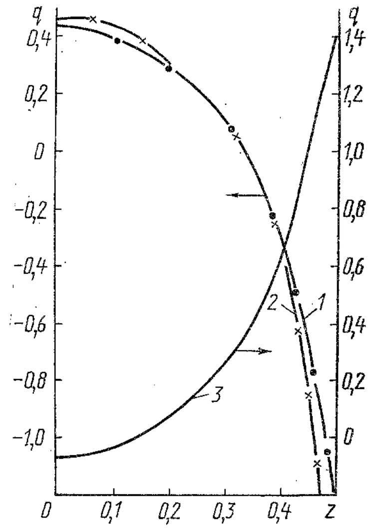

Fig. 2. Thermal flux density and heat liberation on hygrometer surface: 1, Eq. (9); 2, Eq. (7); 3, Eq. (10), q, kW/m².

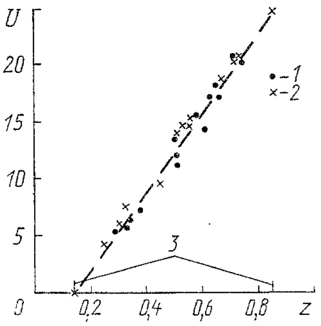

Fig. 3

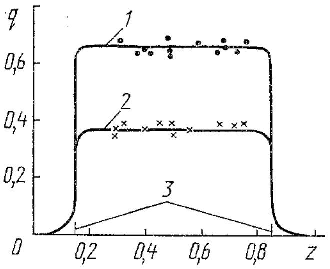

Fig. 4

Fig. 3. Voltage drop in interelectrode gap of lithium chloride sensor DV-1K: 1) t_a = 46°C, P_n = 41.33 mm Hg; 2) 22 and 7.6; 3) electrode boundaries.

Fig. 4. Heat liberation density interelectrode gap of DV-1K sensor, reconstructed from experimental data of Fig. 3; notation as in Fig. 3.

The thermal flux density distribution obtained on the hygrometer surface permits definition of the heat liberation density in its surface layer:

$$q(z) = \alpha\,[t(R,\ z) - t_a] - q(R,\ z), \tag{10}$$

where α is the heat liberation coefficient, t_a is the temperature of the surrounding air. Figure 2 also shows the heat liberation curve in the hygrometer surface layer for the case where the surface temperature is specified by Eq. (4) (see Fig. 1), α = 12 W/(m²·K), t_a = 20°C. It is evident that the heat liberation curve obtained for the hygrometer surface on the basis of the experimental temperature data of [5] has a characteristic uniqueness – the practical absence of heat liberation in the free interelectrode space. This fact is of interest because it can easily be verified experimentally in a regular production sensor.

An experimental test of heat liberation distribution over the height of the interelectrode gap was performed by probing the voltage drop over the height of the gap, using an electrode 0.05 mm in diameter. The hydrometers used were three DV-1K sensors, operating at surrounding medium temperatures of 46 and 22°C and relative humidities of 60 and 39% respectively, with hygrometer supply voltage of 25 V. The experimental results are shown in Fig. 3 in the form of graphs of voltage vs coordinate in the interelectrode gap. The potential drop distribution over the hygrometer interelectrode gap permits reconstruction of the heat liberation distribution therein (Fig. 4).

To determine the hygrometer surface temperature from the experimentally determined heat liberation density in the interelectrode gap (Fig. 4) we make use of the solution of the boundary problem for thermal conductivity equation (1) with temperature field symmetry conditions (2) and (3) and a boundary condition of a third sort on the surface

$$-\lambda \frac{\partial t(r,\ z)}{\partial r}\bigg|_{r=R} = \alpha\,[t(R,\ z) - t_{\mathrm{a}}] - q(z). \tag{11}$$

The solution of this boundary problem, obtained by separation of variables, has the form

$$t(R,\ z) = \frac{2}{h}\sum_{n=0}^{\infty} \frac{\int\limits_0^h [q(z) + \alpha t_{\mathrm{a}}]\cos\frac{n\pi}{h} z dz}{\frac{\lambda n\pi}{h} I_1\left(\frac{n\pi}{h}R\right) + \alpha I_0\left(\frac{n\pi}{h}R\right)} \times$$

$$\times I_0\left(\frac{n\pi}{h}R\right)\cos\frac{n\pi}{h}z. \tag{12}$$

We note that in contrast to the series of Eq. (7) the infinite series obtained here is rapidly converging. Results of a hygrometer surface temperature calculation are shown in Fig. 5.

Comparison of temperature graphs determined by extrapolation of the results of [5] (Fig. 1) with those determined directly from experimentally measured heat liberation density (Fig. 5) shows their qualitative difference. The simplicity and reliability of multiple measurements of potential difference in the interelectrode gap permits us to attribute this qualitative difference to inadequacy of the model used in [5].

Of special interest is the fact that despite the absence of electrode temperature elevation relative to the mean integral hygrometer surface temperature, there is an increase in hygrometer temperature above the temperature of three-phase equilibrium for the corresponding salt (Fig. 5). In our opinion a simple and convincing explanation of this phenomenon is thermodiffusion separation of the components of the air—water vapor mixture in the hygrometer zone, due to temperature gradients which are unavoidable in hydrometer operation. Despite the fact that it has been studied experimentally and theoretically for a large number of gas mixtures [9] the phenomenon of thermodiffusion remains practically uninvestigated for air—water vapor mixtures. An exception is [10], which presented calculated values of the dimensionless thermodiffusion coefficient for this mixture, obtained by extrapolation of experimental data for an H_2—H_2O mixture and having in the authors' opinion, a possible error of 100%. Nevertheless, by using the data of [10] one can quantitatively reconcile (to the accuracy of order of magnitude) the data obtained on temperature nonuniformity in the interelectrode gap and known data on elevation of the hygrometer equilibrium temperature above the three-phase equilibrium temperature of the corresponding salt.

A more detailed quantitative analysis of hygrometer temperature elevation above the three-phase equilibrium temperature will require reliable experimental data on thermodiffusion separation in moist air.

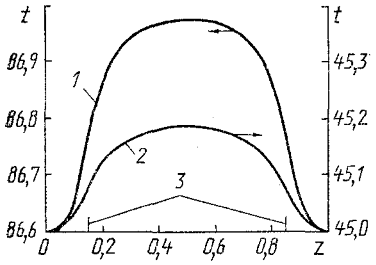

Fig. 5. DV-1K sensor surface temperature calculated by Eq. (12) on basis of heat liberation density data (see Fig. 4); notation as in Fig. 3.

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MOTION OF A GRANULAR MATERIAL OVER A FIXED PLANE

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and M. Balbe

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It is shown that there exists an excess viscous friction produced by shear flow of a continuous medium in the gap between the moving particle layer and the slip plane.

Introduction. In the motion of a granular material on a fixed plane the Coulomb friction law is not always obeyed. This occurs to an extreme degree in the motion of thin layers of powdered materials. It is logical to propose the existence of additional viscous forces caused by shear flow of a continuous medium in the gap between the moving particle layer and the slip plane. This study will evaluate this force.

According to the physical model to be used, we will represent the tangent stresses on the slip surface in the form of a sum of Coulomb and viscous friction

$$\tau = \sigma f + \mu \frac{v}{h}. \tag{1}$$

Neglecting the friction between the outer surface of the material layer and the continuous medium and assuming constancy of particle velocity over layer thickness, which is fully justified for thin layers where the coefficient of external friction of the material is less than the coefficient of internal friction, we write the equation of conservation of momentum in projections in the slip plane, inclined to the horizontal at an angle α:

$$\rho\delta \frac{dv}{dt} = \rho\delta g \sin\alpha - \rho g \delta \cos\alpha\, f - \mu \frac{v}{h}. \tag{2}$$

The layer thickness can be found from the equation of conservation of mass

$$Q = v\delta B\rho. \tag{3}$$

It follows from Eqs. (2) and (3) that

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